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MODEL PREDICTIVE CONTROL FOR GEOTHERMAL BOREHOLE DEPTH DETERMINATION

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Abstract
Ground-source heat pumps provide stable and reliable heating and cooling when designed properly. The confounding effect of the borehole depth for a GSHP system, however, is rarely taken into account for any optimization: the determination of the borehole depth usually comes prior to the selection of corresponding system components and thereafter any optimization of the GSHP system. The depth of the borehole is important to any GSHP system because the shallower the borehole, the larger the fluctuation of temperature of the near-borehole soil temperature. This could lead to fluctuations of the coefficient of performance (COP) for the GSHP system in the long term when the heating/cooling demand is large. Yet the deeper the boreholes are drilled, the more the drilling cost and the operational expenses for the circulation. A controller that reads different building load profiles, optimizing for the smallest costs and temperature fluctuation at the borehole wall, eventually providing borehole depth as the output is developed. Aside from a few scenarios of different weighting factors, the resulting system costs from a MPC for optimizing the borehole depth was verified to maintain the temperature fluctuation at the borehole wall within acceptable ranges also for lengthened scale of time, indicating that the MPC is adequate to optimize for the investment as well as the system performance for various outputs.

Keywords:
Model Predictive Control; GSHP; borehole drilling

1 INTRODUCTION

The depths a geothermal well has to reach to achieve relatively steady temperature is considered to be roughly 3 to 5 feet [1]. It is also understood by expert drillers and designers that heat stored in the ground is depleted if the borehole is too shallow [2]. Yet drilling deeper costs money, which mounts up to a significant number for many considering home improvement [3]. Therefore, designers for geothermal boreholes often need to estimate the best depth for a borehole for a given building [4]. Despite Eskilson's analysis that the axial effect of boreholes becomes significant after a given time period, there is very little optimization that can be done for this process [5]. The existing studies focused either on optimizing an existing system of its heat transfer within boreholes [6], optimizing an arrangement of boreholes [7], or doing the entire optimization process in TRNSYS with predefined system components [8]. This lack of optimization of design for GSHP could be attributed to the complexity of the nonlinear nature of GSHP and the multiple components in, and can be added to them. Some simulation and optimization of GSHP was founding the work of Alavy et al. [9] and Spitler et al. [10], where the seasonal effect of the boreholes are assessed for a borehole design of the components involved in the system, yet again, the depth of the boreholes was not considered as an optimizable component.

Encouraged by such caveat, this paper sets out to investigate the optimization of borehole depths respect to building load and costs involved. The constructed model is inspired mainly a modelling method review by Monzo [11], as well as a few other studies where the seasonal effect of boreholes is abstracted and computed to obtain maximum performance [12]. In this paper, the

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constructing a control problem using a conventional optimized control approach and a model predictive control approach. The outputs from both models are analysed and evaluated to be determine their feasibility and used to assess their potential of being used to assist GSHP design.

2 SYSTEM BASICS

2.1 Axial effects for borehole design

The change in temperature at a given location and time due to the effect of a point source releasing q' units of heat per second was defined as:

$$\varDelta T(r,t) = \frac{q'}{4\pi k_s r} erfc\left(\frac{r}{2\sqrt{(\alpha t)}}\right) \qquad (1)$$

where q'=q/H and q is the rate of heat transfer into or away from the ground(W), k_s is the borehole thermal conductivity (W/mK), and α is the thermal diffusivity of the ground. Another equation used to assess the impact of the injection with respect to the different depths of the system is

$$\varDelta T(r,t,z,H) = \frac{q'}{4\pi k_s H}\left(\frac{erfc\left(\frac{d(u)}{2\sqrt{(\alpha t)}}\right)}{d(u)} - \frac{erfc\left(\frac{d\prime(u)}{2\sqrt{(\alpha t)}}\right)}{d\prime(u)}\right) \qquad (2)$$

in which $d(u) = \sqrt{(r^2 + (z-u)^2}$ and $d'(u) = \sqrt{(r^2 + (z+u)^2}$, with with z being the elevation of the point where the computation is completed. To perform the OCP construction for dynamic optimization, an alternative form of Equation (1) is identified where the exponential integral is simplified for computational purposes:

$$\varDelta T(r,t) = \frac{q'}{4\pi k_s H}\left(ln\frac{4\alpha t}{r^2} - \lambda\right) \qquad (3)$$

where λ is Euler's constant = 0.5772. For the benefit of computing, the outputted average temperature difference along the borehole wall from Equation (3) is selected to be the appropriate component to be included in the optimized control problem design.

2.2 Building load

To better assess the preferred borehole depths throughout an entire year, a synthetic building load profile is selected as in Equation (4):

$$Q(t) = A - Bcos\left(\frac{t}{8760}2\pi\right) - Ccos\left(\frac{t}{24}2\pi\right) - D\cos\left(\frac{t}{24}2\pi\right)\cos\left(\frac{2t}{8760}2\pi\right) \qquad (4)$$

As was described by Marcotte et al. [9], a scenario with A = -30, B=100, C= 50, D = 24 is selected in accordance with the heating-dominated climate profile of Princeton where this study is conducted.

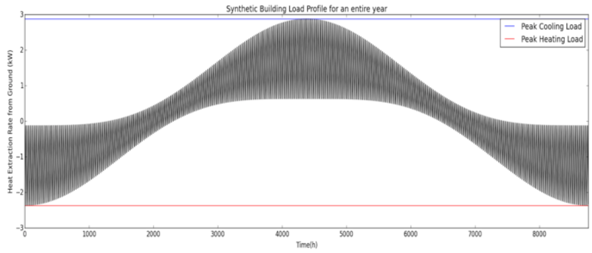

Fig. 1: Synthetic Building Load Profile.

Evaluating the load profile on a yearly basis for the different depths was used with the ΔT function, the trade-off between the different borehole depths and the fluctuation of temperature in the boreholes.

Pronounced fluctuations in the range between 0 and 50 meters of depth should be avoided for houses that are interested in geothermal systems since this could indicate inadequate system performance. But other factors, including the drilling and electricity cost has to be taken into account to optimize for a desirable design.

2.3 First and Electricity Cost

More than 80% of the first cost of geothermal system went to drilling costs [13], many companies even gave quotes of systems based only on the depths of the boreholes. A simplification is therefore necessary as described by Rafferty [14], that the cost of wells that aren't deeper than 500ft has a hard drill cost of 5$ per feet borehole per inch diameter. For a borehole with 0.075 m radius (common according to Monteyne et al., that is 48.425 dollar per meter for the drilling [15].

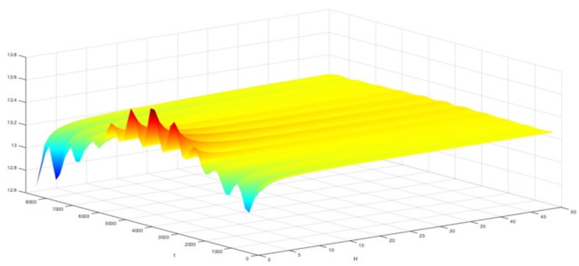

Fig. 2: Synthetic load profile for the building for 8760 hours applied with a depths profile from 0 to 50 meters.

The electricity cost of the system is split into the power consumption of the heat pump, which is related with the building load as $P_{hp} = \frac{Q_{building}}{COP}$, in which COP is a variable that with respect to the inlet water temperature of the heat pump and have heating and cooling modes. Also included is the pumping costs that is a variable of the depth of borehole, as can be obtained from the pumping power equation as $P_{electricity} = \frac{\dot{m}\rho g h}{3.6\times10^6}$, in which $\dot{m}$ is the flow capacity is in m^3/h while ρ is the density of

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$P_{electricity} = \frac{\dot{m}\rho g h}{3.6\times10^{6}}$, in which $\dot{m}$ is the flow capacity is in m^3/h while ρ is the density of circulating fluid and g is the gravitational constant at 9.81 m/s.

3 CONTROL PROBLEM FORMULATION

3.1 Modified Cost function

The aim is to optimize the borehole depth such that both the monetary cost and ground temperature stabilization can be achieved at the same time. As those two are conflicting objectives, a multi-objective approach is put forward. The cost function, presented by Equation (5) is a weighted sum of the drilling cost Jd, electricity cost Jc and the thermal fluctuation cost JT , evaluated over a time period of [0, teval], which is take as an entire year for this control problem formulation.

$$J_{tot} = (1-K)\left(\int_0^{t_{eval}} J_c dt + J_d\right) + K\int_0^{t_{eval}} J_T dt \quad (5)$$

For K = 1, the optimal control profile minimizes the temperature fluctuation and assumed no consideration for the cost is needed, while for K = 0, the optimal control profile maximizes the economic investment required for the system while ignoring the possible decrease of geothermal potential. $J_T = \Delta T^2$ represents the squared temperature penalty, while similarly $J_e = c_{el}(P_{hp}(t) + P_{cl})$, and the expression J_d represents the cost resulting from the drilling.

Model Predictive Control is known as a class of computer algorithms that predicts future system behaviour while computing the correct control actions required to drive the predicted output towards the desired output. Therefore, to modify the existing controller to form an MPC problem the cost function will first have to be modified into a multivariable control algorithm. This requires an internal dynamic model of the process, a history of past control moves, and an optimization cost function over the receding prediction horizon in order to calculate the optimum control move [13]. This, combined with the aim of this study, provide us the following modified cost function:

$$J_T = \Delta T(t,H,r,Q_b) + \sum_i^{t-1} \Delta T(i,H,r,Q_b(i-1)) \quad (6)$$

3.2 Controller Model

The system dynamics are defined by a set of ordinary differential equations. They correspond to a simplified setup of a geothermal water-to-water system of a varied depth profile:

$$\dot{Q}_b = \frac{25}{6}\pi\sin\left(\frac{\pi t}{12}\right) + \frac{5}{219}\pi\sin\left(\frac{\pi t}{4380}\right) + \frac{25}{12}\pi\sin\left(\frac{\pi t}{12}\right)\cos\left(\frac{\pi t}{2190}\right) + \frac{5}{438}\pi\sin\left(\frac{\pi t}{2190}\right)\cos\left(\frac{\pi t}{12}\right) \quad (7)$$

$$\dot{P_{hp}} = \frac{\dot{Q_b}}{COP} \quad (8)$$

$$\Delta T(r,t) = \frac{\dot{Q_b}}{4\pi k_s H}\left(ln\frac{4\alpha t}{r^2} - \lambda\right) - \frac{\dot{Q_b}}{4\pi k_s H}\left(\frac{r^2}{4\alpha t}\right) \quad (9)$$

$$C_i\dot{T_i} = \dot{m}(T_r - T_i) + \dot{Q_b} \quad (10)$$

In the dynamic equations, the terms are correspondingly representing Qb building load(kW),Php as the power consumption of the heat pump, COP as the coefficient of performance, Ti as the incoming water temperature for the GSHP in ◦C, and Ci represented the thermal capacity of the water (J/K). In this formulation the control variable is the depth of the borehole H. Modulating H with respect to the cost function will allow for the minimum amount of economic input and the least depletion of the geothermal resources, or more specifically in the form of the following equations:

$$J_d = 48.425H \quad (11)$$

$$P_{cl} = 0.266\frac{kg}{s}\times3600\frac{kg}{h}\times1000\frac{kg}{m^3}\times9.81\frac{m^2}{s}\times H = 2.609H \quad (12)$$

$$COP_{heating} = \frac{T_i}{T_i - T_r} \qquad COP_{cooling} = \frac{T_r}{T_r - T_i} \quad (13)$$

3.3 Solving with Initial, Boundary Condition and temperature constraints

Corresponding to the dynamic ODEs, the model states are respectively $S = [Q_b, P_{hp}, COP, T_i]$ .While the other three parameters are all bounded to T_i with it being the water supplied by the ground, it is determined that it should be a bounded value between t0 and teval since the optimization process was for an entire year, such that $T_i(t_0) = T_i(t_{end})$. The upper and lower bounds of the inlet temperature are determined according to empirical values as 37◦C and 7.2◦C as according to [2]. The continuous optimal control problem has to be discretized for a direct numerical solver. The control variable is discretized using a piece-wise constant function, such that during one discretization time step H is constant. The smaller the discretization time step, the smaller the difference between the continuous and discretized optimal control profiles, the longer the computation time.

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The formulations discussed adopted a discretization time step of one hour, yielding a total 8670 control variables to be optimized. The Automatic Control and Dynamic Optimization toolkit ACADO is used for this study to discretize the OCP with multiple shooting methods developed by Bock and Plitt: state variables are discretized at a different time step than the control variables to minimize the discretization error, for which a standard Runge-Kutta 45 integrator was used. The resulting discrete-time optimization problem is solved with the following trajectory:

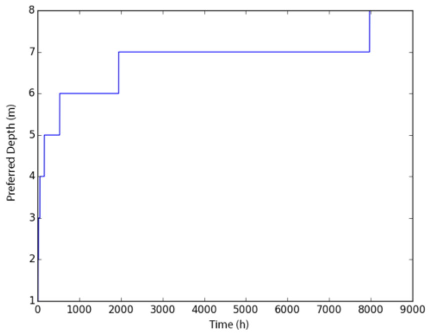

Fig. 3: Trajectory of obtaining preferred depth for heating-dominated season.

3.4 Solving with Initial, Boundary Condition and temperature constraints

In lack of a better solver, the MPC problem was constructed in Python where the search of minimized cost function was dealt with nested for loops. The initial condition for each time step is initialized for every one of the 8760 iterations. The for loops also made it possible for the temperature penalty to be carried on for the next time step - that is, an increment of t = 1 in the Python function. The resulting temperature fluctuation at the borehole can be found in Fig.4 as the followings: The output is a borehole depth of 1 meters of an equal weighting between the cost and the temperature penalty for this simulation. Correspondingly, the overall system cost for a year was found to be $8397.96 with a temperature penalty of -0.41 °C, this is consistent with the findings from Garber et al. where the operational costs were at the same order of magnitude [16].

3.5 Performance Evaluation

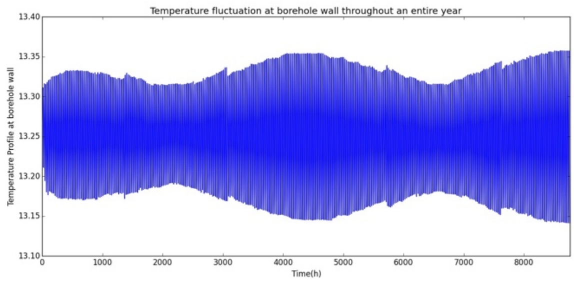

Fig. 4: Trajectory of the temperature profile at borehole wall during year-long simulation.

Various weighting factors K (0.1-0.9) were used to assess the relationship between the objectives shown in the following Fig. 5. The two plateaus that can be observed in Fig. 5 are in fact compensated by the increased fluctuation at borehole wall level.

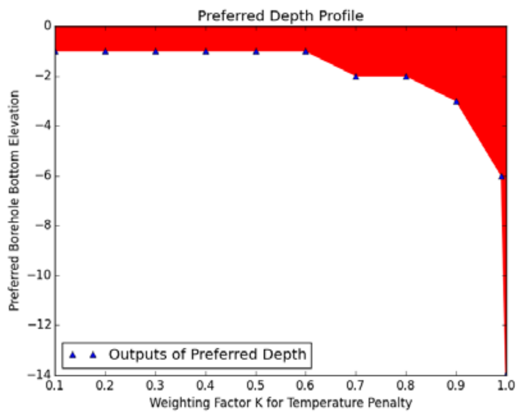

Fig. 5: Preferred variation of depths with respect to different weighting factor.

The average temperature drop at borehole walls can also be obtained in the following Fig. 6 as well as the resulted cost in Fig. 7.

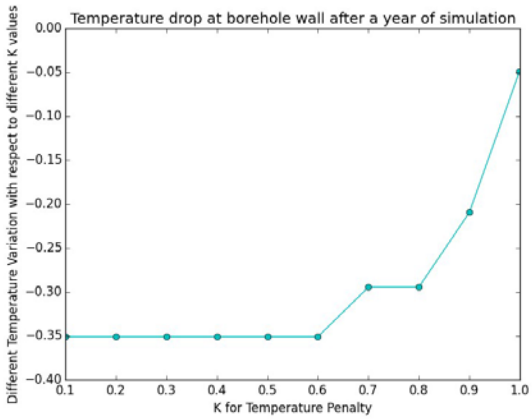

Fig. 6: Mean temperature variation at borehole walls at end of each simulation year for different weighting factor K.

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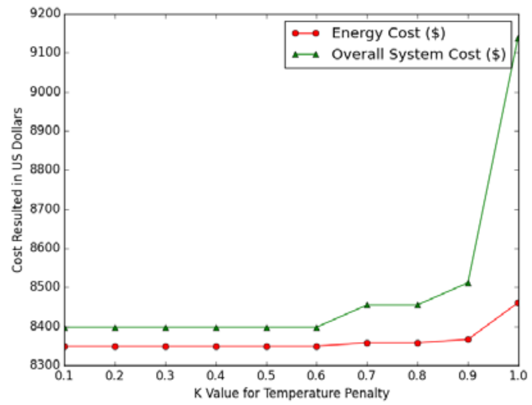

Fig. 7: Resulting costs (in US$) variation resulting from different weighting factors.

3.6 Expanding the time frame

The performance of the controller is further tested by expanding the time frame of simulation where the resulting temperature fluctuation throughout simulation can be seen in the following plots:

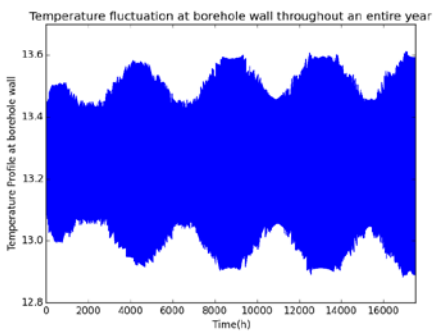

Fig. 8: Temperature fluctuations at borehole walls from a 2-year simulation.

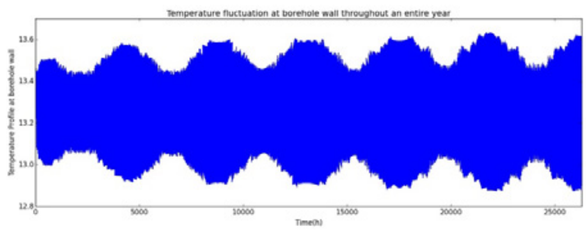

Fig. 9: Temperature fluctuations at borehole walls from a 3-year simulation.

4 CONCLUSION AND DISCUSSIONS

The possibility to develop an easily accessible optimization tool for a given building profile was investigated in this paper. Both the dynamic optimal system control problem and the model predictive control methods were used to seek the optimal borehole depth. Model predictive control was determined to be more adequate in that it allows the temperature penalty to be carried from one-time step to another.

It is believed that this MPC-based optimal design estimator should be able to help geothermal system designers to make quick and efficient decisions for different kinds of climate and load profile. Since the MPC controller was built in Python, it is easily convertible to a weather-file based borehole depth design optimizer. It can include variables such as the electricity and weighting factors that are easily adjustable. Although from the results of this paper it is advised that any subjective weighting aim for deeper for the benefit of long-term system performance.

Further research could be focused on introducing more drastically different load profiles, and include increased control logic to introduce further components - solar-hybrid system or thermal storage system, to be optimized for preferred borehole depth, etc. Lengthening the simulated time period could also be feasible to more appropriately examine the costs since the resulting effect of temperature variation will further lead to further changes in the heat pump performance.

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