Bending-Active Plates

Form and Structure

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Abstract

Aiming to support the current research on bending-active plate structures, this paper discusses the topic of form-finding and form-conversion and presents examples to illustrate the formal and structural potential of these design strategies. Following a short introduction into the topic, the authors reflect on the specific challenges related to the design of bending-active plate structures. While previous research has mainly focussed on a bottom-up approach whereby the plates first were specified as basic building blocks and the global shape of the structure resulted from their interaction, the main emphasis of this paper lies on a possible top-down approach by form-conversion. Here, the design process starts with a given shape and uses surface tiling and selective mesh subdivision to inform the geometrical and structural characteristics of the plates needed to assemble the desired shape. This new concept entails some constraints, and the paper therefore provides an overview of the basic geometries and mechanics that can be created by following this approach. Finally, to better demonstrate the innovative potential of this top-down approach to the design of bending-active plate structures, the authors present two built case studies, each of which is a proof of the concept that pushes the topic of form-conversion in a unique way. While the first one takes advantage of translating a given shape into a self-supporting weave pattern, the second case study gains significant structural stability by translating a given form into a multi-layered plate construction.

Keywords:
bending-active structures, elastic bending, plate structure, form-finding, nonlinear analysis

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1. Introduction

With the rise of new simulation strategies and computational tools, a new generation of architects and engineers is getting more interested in form-finding architectural systems. The key motivation of this approach is to determine a force equilibrium to generate and stabilise a structure just by its geometry. While the membrane and shell structures of pioneers like Frei Otto, Heinz Isler, and Felix Candela were often derived from model-based form-finding processes or using pure geometrical bodies (Chilton 2000, Otto 2005, Garlock & Billington 2008), today's structures often arise from advanced digital simulations and the integration of material behaviour therein (Adriaenssens et al. 2014, Menges 2012).

A good example for the new possibilities emerging from a physics-informed digital design process is the research done on bending-active structures. This type of structural system uses large-scale deformations as a form-giving and self-stabilising strategy (Knippers et al. 2011, Lienhard et al. 2013, 2014, Schleicher et al. 2015). Typically, bending-active structures can be divided into two main categories, which relate to the geometrical dimensions of their constituent elements. For instance, 1D systems can be built with slender rods and 2D systems out of thin plates (Fig. 1). While extensive knowledge and experience exists for 1D systems, with elastic gridshells as the most prominent application, plate-dominant structures have not yet received much attention and are considered difficult to design. One reason is that plates have a limited formability since they deform mainly along the axis of weakest inertia and thus cannot easily be forced into complicated geometries.

However, this obstructive limitation of the smallest building block can also be understood as special advantage. Used strategically, it offers not only more control over the global formation process, but can also be used to inform the individual parts of the assembled structure based on the features of the overall shape. This essentially means that form-finding in the context of bending-active structures could evolve from a bottom-up to a top-down approach, starting with a desired global shape first and then solving the form-force equilibrium of its parts. Following this approach renders the ability to construct a given shape by integrating local bending of its components while guaranteeing that stresses remain within the permitted working range of the material.

2. Typical Design Approaches and Resulting Challenges

Bending-active structures are often designed by following either a behaviour-based, geometry-based, or integrated approach (Lienhard et al. 2013). While the first category refers to traditional, intuitive use of bending during the construction process and

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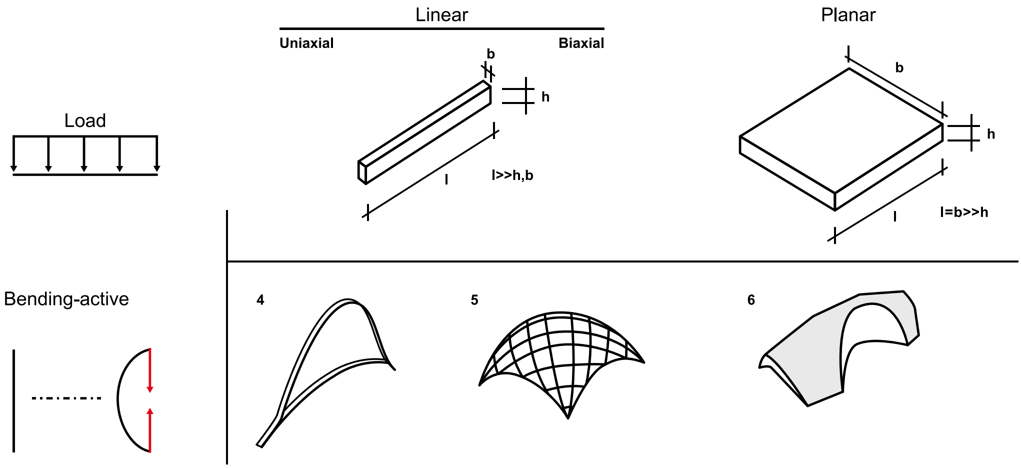

Figure 1. Classification of bending-active structures based on the member's geometrical dimension (from Knippers et al. 2011).



Figure 2a. Traditional Mudhif reed house.
Figure 2b. Plydome.
Figure 2c. ICD/ITKE Research Pavilion 2010.

relies only on hands-on experience regarding the deformation behaviour of the used building material (Fig. 2a), the latter two categories describe a more scientific take on the design of bending-active structures. Here, experimental and analytical form-finding techniques were conducted beforehand and then informed the design process.

One example for bending-active plate structures that were built by following a geometry-based design approach are Buckminster Fuller's plydomes (Fuller 1959). This construction principle is based on approximating the basic geometry of a sphere with a regular polyhedron. Its edges and angles are then used to arrange multiple plates into a spatial tiling pattern, which is fastened together by bending the plates at their corners (Fig. 2b). The resulting structure is made out of identical plates joined together by placing bolts at predefined positions. Even though this technique allowed Fuller to construct a double-curved spherical shape out of

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initially planar and then single-curved plates, this methodology also had several shortcomings. First and foremost, it is limited to basic polyhedral shapes. Only because of the repetitive angles was it possible to use identical plates. Furthermore, at his time Fuller was forced to compute the needed overlap of the plates and the exact position of the pre-drilled holes mathematically. The only way to calibrate this data was by producing plydomes in series and improving the details over time.

Compared to that, following an integrative design approach for bending-active plate structures provides more flexibility and renders the opportunity for computational automatisation. A prominent example is the 2010 ICD/ITKE Research Pavilion (Fig. 2c). As characteristic for an integrative approach, this project started with intensive laboratory testing to better understand the limiting material behaviour of the used plywood. The results of these physical experiments were then integrated as constraints into parametric design tools and used to calibrate finite-element simulations. Synchronising physical and digital studies ensured that the form-finding techniques provided an accurate description of the actual material behaviour while at the same time giving more feedback on the resulting geometry of the structure. This project even went so far to re-create the actual bending process by simulating the deformation of every strip into a cross-connected and elastically pre-stressed system (Lienhard et al. 2012).

While the last project is definitely innovative, it should be pointed out that the integrated approach here was used mainly in a bottom-up way and thus narrowed the possible design space. For the future development of bending-active plate structures, however, it may be desirable to prioritise a top-down approach, which gives more weight to the target geometry and thus more freedom to the designer. However, the key challenge remains and boils down to how to assess both the global shape as well as the local features of the constituent parts for structures in which geometrical characteristics and material properties are inevitably linked together and similarly affect the result.

3. Form Conversion

The principal limit to the formal potential of bending-active structures lies in the restrictions on the material formability. The only deformations that can be achieved within stress limits are the ones that minimise the stretching of the material fibres. For strips and plate-like elements, these reduce to the canonical developable surfaces: cylinders and cones. Attempting to bend a sheet of material in two directions will either result in irreversible, plastic deformations or ultimately failure. Such a strict requirement severely limits the range of structural and architectural potential for plate-based bending-active systems. To expand the range of achievable shapes, it is therefore necessary to develop workarounds

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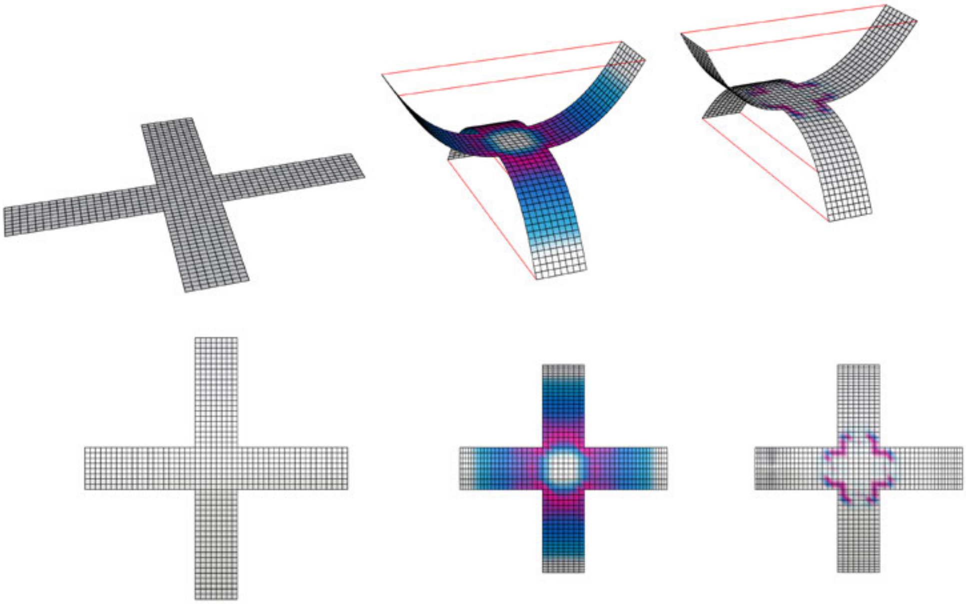

Figure 3a. Multidirectional strip.
Figure 3b. von Mises stress distribution after bending.
Figure 3c. Gaussian curvature.

for the induction of Gaussian curvature. To overcome such limitations, multidirectional bending can be induced by strategically removing material and freeing the strips from the stiffening constraint of the surrounding. A similar approach is presented by Xing et al. (2011) and referred to as band decomposition. The key principle is illustrated in Figure 3.

Here a continuous rectangular plate is reduced to two orthogonal strips. The strips are later bent into opposite directions in a finite element simulation using the ultra-elastic contracting cable approach based on Lienhard, La Magna and Knippers (2014). The bending stiffness of the plate, depending proportionally on its width b, results in a radical increase of stiffness in the connecting area between the strips. As a result, the connecting area almost remains planar, and therefore the perpendicular bending axis remains unaffected by the induced curvature. In this way it becomes possible to bend the strips around multiple axes, spanning different directions but still maintaining the material continuity of a single element. Figure 3b depicts the resulting von Mises stresses calculated at the top fibres. The gradient plot clearly displays an area of unstressed material at the intersection between the two strips, as expected based on the previous arguments. A local stress concentration appears at the junction of the strips due to the sharp connecting angle and inevitable geometric stiffening happening locally in that area.

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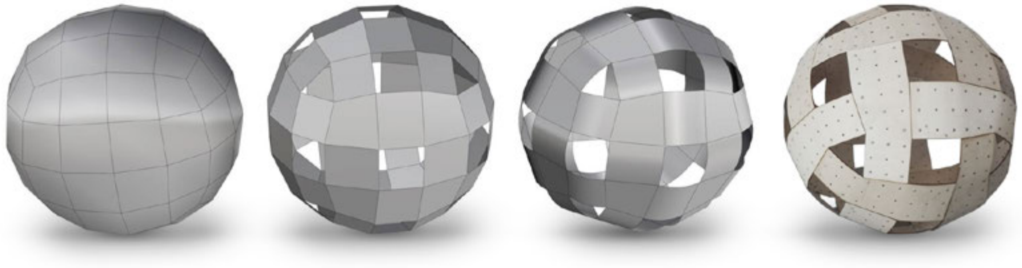

Figure 4a. Mesh of target surface.
Figure 4b. Offset and edge bridging.
Figure 4c. Bending of bridging elements.
Figure 4d. Plywood prototype of sphere test case.

This result can be compared with Figure 3c, which displays the Gaussian curvature of the bent element. From the plot it is clear that the discrete Gaussian curvature (based on Meyer et al. 2003) of the deformed mesh is everywhere zero, apart from a small localised area at the intersection of the two branches. This confirms that, within stress limits, flat sheets of inextensible material can only be deformed into developable surfaces at most.

Based on the strip approach defined so far, the general procedure for an arbitrary freeform surface is summarised in the following steps:

1. Mesh the target surface (Fig. 4a).
2. Perform an interior offset for each face of the mesh.
3. Connect the disjointed faces by creating a bridging element; two faces initially sharing an edge will be connected (Fig. 4b).
4. The bridging element is modified to take into account the bending curvature. Assuming that the start and end tangent plane of the bridging element coincide with the surfaces to be connected, the element can be defined through a simple loft (Fig. 4c).
5. Unroll the elements.

The presented method maintains general validity for any arbitrary source mesh. In the case of an Ngon mesh, its banded dual will have strips with N arms departing from the centre surface element. The geometry of the voids is defined by the valence of the mesh. For the sphere example a 4-valent source mesh produces square voids throughout the banded structure. A tri-valent hexagonal mesh would produce triangular voids and so forth.

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4. The Geometry and Mechanics of Bending-Active Plate Structures

It is typical in engineering to distinguish between plate and shell structures, the main difference being that plates are initially flat and shells already present curvature in their stress-free configuration. Both structures can be identified as having thickness significantly smaller than length and width. In this way the geometry of a shell or plate is uniquely defined by their centre surface and local thickness (Bischoff et al. 2004). The structural behaviour of shells and plates is characterised by two main states of deformation, membrane and bending action. Membrane deformations involve strain of the centre surface, whilst bending dominated deformations roughly preserve the length of the mid-surface fibres. Under bending, only the material fibres away from the mid-surface are fully exploited, therefore the structural elements are much more flexible. Pure bending deformations are also called *inextensional deformations* as the neutral surface is completely free from longitudinal extension or compression. In mathematical terms, a transformation that preserves lengths is also referred to as an *isometry*. *Pure bending*, *inextensional*, and *isometric deformations* are all synonyms that are often used interchangeably in literature, preferring one term over another to highlight either mechanical or mathematical aspects. Strictly speaking, the only isometric transformations of the plane are into cones and cylinders, i.e. developable surfaces.

In structural applications, membrane deformation states are generally preferred as the cross-section is completely utilised and the load-bearing behaviour of the shell is significantly enhanced. On the other hand, characteristics of inextensional deformations may be exploited in certain situations, for example, deployable or tensile structures which might benefit from bending dominated transition stages. In the context of bending-active structures, inextensional deformations represent the main modality of shape shifting, as the bending elements may undergo large deformations without reaching a critical stress state for the material. Owing to the high flexibility of thin plates with respect to bending, this state of deformation may be regarded as the dominating mechanical effect for bending-active structures as having the strongest effect on the nonlinear behaviour of plates.

The relationship between large deformations and pure bending is well understood in light of energetic arguments explained in the following paragraph. An important assumption in the context of bending-active structures is that of a perfect elastic response of the material. This is the case of Hookean elasticity, which assumes a linear elastic response of the material and therefore yields a proportional relationship between strain and stress (Audoly & Pomeau 2010). This assumption is valid for small strains in general, which is commonly the case for bending-active structures. In the membrane approximation, the elastic energy of a plate reads:

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(4.1)

$$\varepsilon_{mb} \sim Eh \iint_P (\epsilon_{\alpha\beta})^2_{cs}\, dxdy$$

where the subscript *'cs'* means that we can evaluate the density of the elastic energy along the centre surface. The approximation (4.1) can be understood as following: It is the surface integral of the squared, 2-dimensional strain along the centre surface $\epsilon_{\alpha\beta}$, multiplied by the factor Eh, which is proportional to the thickness h and to Young's modulus E of the material.

Conversely, the bending energy of a 2-dimensional plate assumes the following form:

(4.2)

$$\varepsilon_b \sim \frac{1}{2} \iint_P \frac{Eh^3}{12} \left(\frac{1}{R_{cs}(x,y)}\right)^2 dxdy$$

which can be read as the surface integral of the squared curvature (dependent on x and y) of the centre surface, multiplied by the factor Eh^3, which is commonly called *bending stiffness*.

Comparing the stretching energy (4.1) with the bending energy (4.2) shows that the small thickness h comes in the flexural energy (4.1) with a larger power than in the stretching energy, i.e. h^3 in place of h. For very thin plates, this makes the energy of isometric deformations much lower than those involving significant stretching of the centre surface. As a result, large deformations occur mainly under bending, as the low energy involved in the process is generally compatible with the strain limits of the material.

Although commonly referred to as bending-active, the term has been specifically coined to describe a wide range of systems that employ the large deformation of structural components as a shape-forming strategy. Besides bending, torsional mechanisms can also be employed to induce form, as the energy involved is of similar order of magnitude compared to bending. An essential requirement for bending-active structures is that the stress state arising from the form-finding process does not exceed the yield strength of the material. Based on the previous assumptions of perfect elastic material response and thin, slender sections, the maximum bending curvature and maximum torsional angle twist can be checked against the following relationships:

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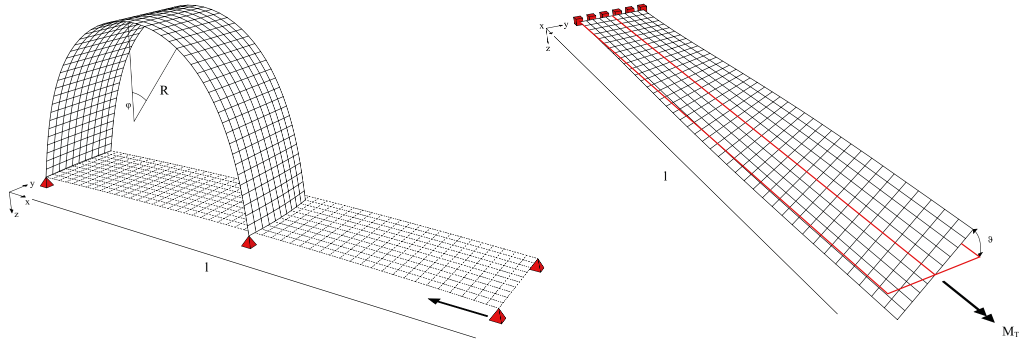

Figure 5a. Material strip subject to axial bending.
Figure 5b. Material strip subject to torsion.

$$k = \frac{d\varphi}{dx} = \frac{M_B}{EI} \qquad \frac{d\theta}{dx} = \frac{M_T}{GJ}$$

$$\sigma_{max} = \frac{M_B}{W} \qquad \tau_{max} = \frac{M_T}{W_T}$$

$$k_{min} = \frac{1}{R_{max}} = \frac{\sigma_{max} W}{EI} = \frac{2\sigma_{max}}{Eh} \qquad \frac{d\theta}{dx} = \frac{\tau_{max} W_T}{GJ} = \frac{\tau_{max}}{Gh}$$

where:

k	curvature [1/m]	θ	angle of twist [rad]
M_B	bending moment [kNm]	M_T	torsional moment [kNm]
E	Young's modulus [N/mm^2]	G	shear modulus [N/mm^2]
I	moment of inertia [m^4]	J	torsional constant [m^4]
$W = bh^2/6$	resistance moment [m^3]	$W_T = bh^3/3$	torsional resistance [m^3]
σ_{max}	max. allowable stress [N/mm^2]	τ_{max}	max. shear stress [N/mm^2]
h	section height [mm]	h	section height [mm]

These equations refer to the classic Euler-Bernoulli model for bending and de Saint-Venant torsion model for beams. Both models ignore higher order effects, respectively deformations, caused by transverse shear and torsional warping. Although generally non-neglectable for large deformations, owing to the previous assumptions of slender cross-sections and elastic behaviour, it is safe to assume these values for a preliminary check of the master geometry.

The complexity of the structural systems and form-finding procedures still require an accurate numerical analysis. In general, currently available simulation tools can be subdivided into two categories. The first one, dynamic relaxation (DR), is a numerical iterative method to find the solution of a system of nonlinear equations. It has been successfully employed in engineering applications for the form-finding of membrane and cable net structures (Barnes 1999, Adriaenssens & Barnes 2001) and in modified versions also for torsion related problems in surface-like shell elements (Nabaei et al. 2013). The second method relies on finite element

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simulation (FEM). Non-linear finite element routines have advanced so much lately that it is becoming more common to integrate them in the design process. All the results presented here were achieved through geometrical non-linear finite element simulations run in SOFiSTiK.

5. Case Studies

The following two case studies are both made out of the same material – 3 mm birch plywood. This plywood consists of three layers and has different mechanical behaviours along the main fibre orientation (stronger) and against it (softer). This is due to the fact that the fibre direction of the upper and lower layers is oriented in one direction and rotated by 90° in the centre layer. Based on this the plywood also has two values for the minimal bending radius that can be achieved as well as two values for the maximum axial twist the material can undergo. The Young's modulus of a 3 mm plywood along the grain is: Em_{II} = 16471 N/mm² and against the grain is: $Em_{\perp}$ = 1029 N/mm².

5.1 Case Study: Berkeley Weave

The first case study investigates the design potential emerging from integrating both bending and torsion of slender strips into the design process. A modified Enneper surface acts as a base for the saddle-shaped design (Fig. 6a). This particular form was chosen because it has a challenging anticlastic geometry with locally high curvature. The subsequent conversion process into a bending-active plate structure followed several steps. The first was to approximate and discretise the surface with a quad mesh (Fig. 6b). A curvature analysis of the resulting mesh reveals that its individual quads are not planar but spatially curved (Fig. 6c).

The planarity of the quads, however, is an important precondition in the later assembly process. In a second step, the mesh was transformed into a four-layered weave pattern with strips and holes. Here, each quad was turned into a crossing of two strips in one direction intersecting with two other strips in a 90-degree angle. The resulting interwoven mesh was then optimised for planarisation. However, only the regions where strips overlapped were made planar (blue areas), while the quads between the intersections remained curved (Fig. 6d). A second curvature analysis illustrates the procedure and shows zero curvature only at the intersections of the strips while the connecting arms are both bent and twisted (Fig. 6e). In the last step, this optimised geometrical model was used to generate a fabrication model that features all the connection details and strip subdivisions (Fig. 6f).

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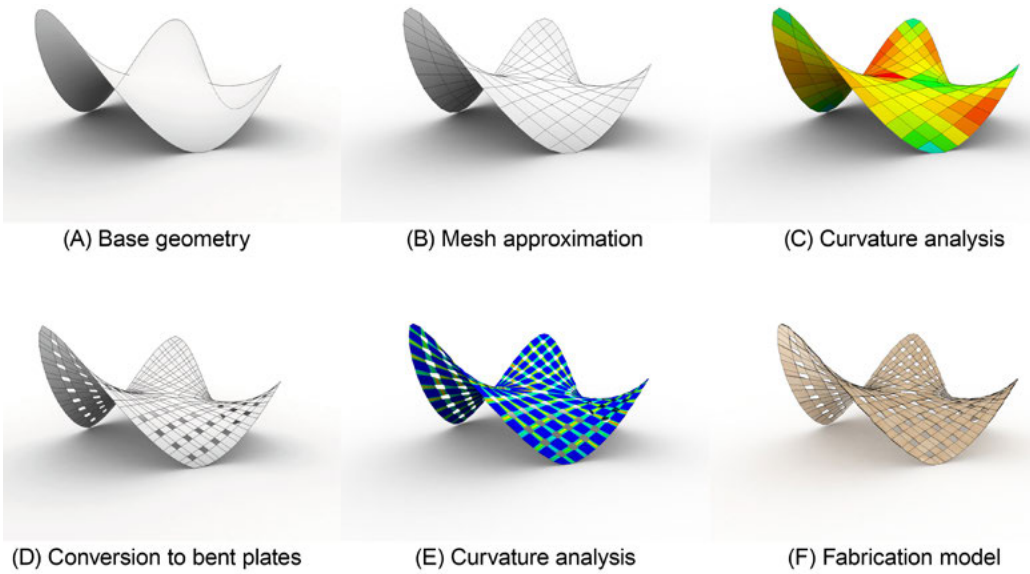
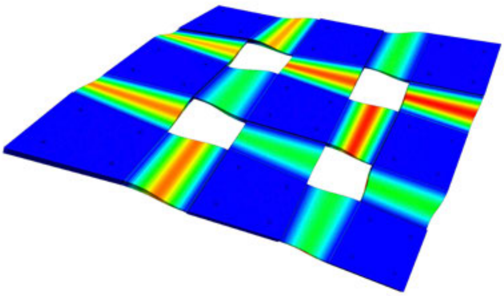


Figure 6. Generation process and analysis.



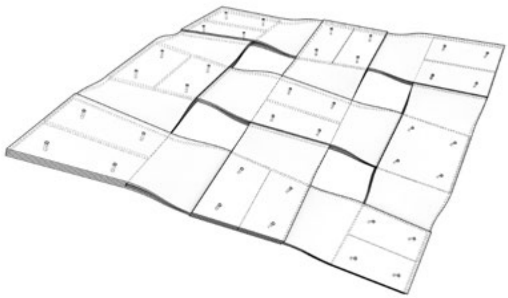


Figure 7a. Analysis of Gaussian curvature.
Figure 7b. Schematic of the weaving and technical details.

A closer look at the most extremely curved region in the structure illustrates the complexity related to this last step (Fig.7a). To allow for a proper connection, bolts were placed only in the planar regions between intersecting strips. Since the strips are composed out of smaller segments, it was also important to control their position in the four-layered weave and the sequence of layers. A pattern was created which guaranteed that strip segments only ended in layers two and three and are clamped by continuous strips in layers one and four. A positive side effect of this weaving strategy is that the gaps between segments are never visible and the strips appear to be made out of one piece. The drawback, however, is that each segment has a unique length and requires specific positions of the screw holes (Fig. 7b).

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To demonstrate proof of concept for this design approach, this case study was built in the dimensions of 4 m x 3.5 m x 1.8 m (Fig. 8). The structure is assembled out of 480 geometrically different plywood strips fastened together with 400 similar bolts. The material used is 3.0 mm thick birch plywood with a Young's modulus of Em_{II} = 16471 N/mm² and $Em_{\perp}$ = 1029 N/mm². Dimensions and material specifications were employed for a finite-element analysis using the software SOFiSTiK. Under consideration of self weight and stored elastic energy, the minimal bending radii are no smaller than 0.25 m and the resulting stress peaks are still below 60% of permissible yield strength of the material.

5.2 Case Study: Bend9

The second case study is a multi-layered arch that spans over 5.2 m and has a height of 3.5 m. This project was built to prove the technical feasibility of using bending-active plates for larger load-bearing structures. In comparison to the previous case study, this project showcases a different tiling pattern and explores the possibility to significantly increase a shape's rigidity by cross-connecting distant layers with each other.

To fully exploit the large deformations that plywood allows for, the thickness of the sheets had to be reduced to the minimum, leading once again to the radical choice of employing 3.0 mm birch plywood. Since the resulting sheets are very flexible, additional stiffness needed to be gained by giving the global shell a peculiar geometry which seamlessly transitions from an area of positive curvature (sphere-like) to one of negative curvature (saddle-like) (Fig. 9a). This pronounced double-curvature provided additional stiffness and avoided undesirable deformation modes of the structure. Despite the considerable stiffness achieved through shape, the choice of using extremely thin sheets of plywood required additional reinforcement to provide further load resistance. These needs were met by designing a double-layered structure with two cross-connected shells.

As in the previous example, the first step of the process was to convert the base geometry into a mesh pattern (Fig. 9b). In the next step a preliminary analysis of the structure was conducted, and a second layer was created by offsetting the mesh. As the distance between the two layers varies to reflect the bending moment calculated from the preliminary analysis, the offset of the surfaces changes along the span of the arch (Fig. 9c). The offset reflects the stress state in the individual layers, and the distance between them grows in the critical areas to increase the global stiffness of the system. The following tiling logic that was used for both layers guarantees that each component can be bent into the specific shape required to construct the whole surface. This is achieved by strategically placing the voids into target positions of the master geometry, as described in Section 3, and thereby ensuring that the bending process can take place without prejudice for the individual components (Fig. 9d). Although initially flat, each

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Figure 8. View of the plywood installation Berkeley Weave.

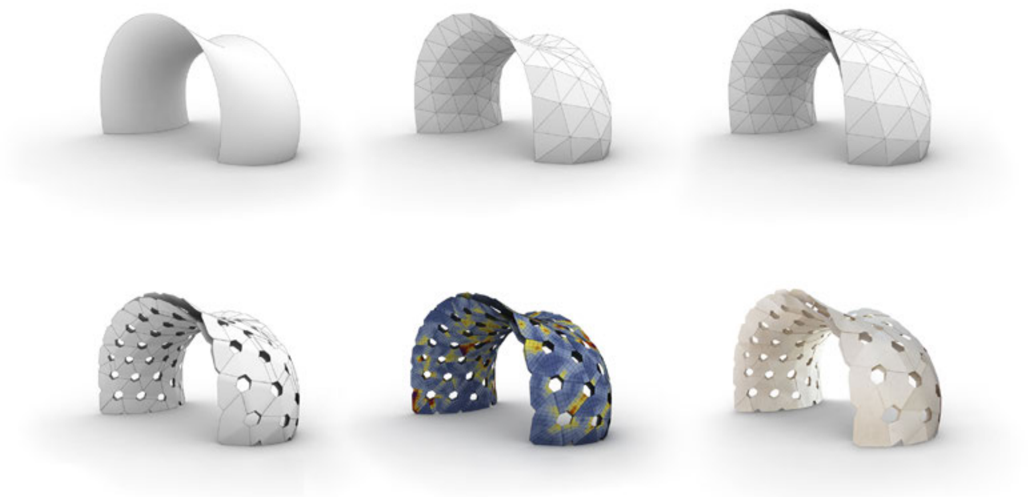

Figure 9a. Base geometry.
Figure 9b. Mesh approximation.
Figure 9c. Double layer offset.
Figure 9d. Conversion to bent plates.
Figure 9e. Finite-element analysis.
Figure 9f. Fabrication model.

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element undergoes multi-directional bending and gets locked into position once the neighbours are added to the system and joined together. The supple 3.0 mm plywood elements achieve consistent stiffness once assembled together, as the pavilion, although a discrete version of the initial shape, still retains substantial shell stiffness. This was validated in a second finite-element analysis that considered both self-weight as well as undesirable loading scenarios (Fig. 9e). Finally, a fabrication model was generated and the structure fabricated (Fig. 10).

The built structure employs 196 elements unique in shape and geometry (Fig. 11a). 76 square wood profiles of 4 cm x 4 cm were used to connect the two plywood skins (Fig. 11b). Due to the varying distance between the layers, the connectors had a total of 152 exclusive compound mitres. The whole structure weighs only 160 kg, a characteristic which also highlights the efficiency of the system and its potential for lightweight construction. The smooth curvature transition and the overall complexity of the shape clearly emphasise the potential of the construction logic to be applied to any kind of double-curved freeform surface.

6. Conclusions

The two built case studies clearly illustrate the feasibility of a construction logic that integrates bending deformations strategically into the design and assembly process. Both structures presented are directly informed by the mechanical properties of the thin plywood sheets employed for the project. Their overall geometry is therefore the result of an accurate negotiation between the mechanical limits of the material and its deformation capabilities.

The assembly strategy devised for both prototypes drastically reduces the fabrication complexity by resorting to exclusively planar components which make up the entire double-curved surfaces. Despite the large amount of individual geometries, the whole fabrication process was optimised by tightly nesting all the components to minimise material waste, flat cut the elements, and finally assemble the piece on-site.

The very nature of the projects required a tight integration of design, simulation, and assessment of the fabrication and assembly constraints. Overall, the Bend9 pavilion and Berkeley Weave installation exemplify the capacity of bending-active surface structures to be employed as a shape-generating process. For on-going research, the buildings serve as first prototypes for the exploration of surface-like shell structures that derive their shape through elastic bending.

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Figure 10. View of the Bend9 structure.

Figure 11a. Detail of the elements.
Figure 11b. Detail of the connecting elements.

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