Marionette Mesh

From Descriptive Geometry to Fabrication-Aware Design

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Abstract

This paper introduces an intuitive method for the modelling of free-form architecture with planar facets. The method, called Marionette by the authors, takes its inspiration from descriptive geometry and allows one to design complex shapes with one projection and the control of elevation curves. The proposed framework only deals with linear equations and therefore achieves exact planarity, for quadrilateral, Kagome, and dual Kagome meshes in real-time. Remarks on how this framework relates to continuous shape parameterisation and on possible applications to engineering problems are made.

Keywords:
structural morphology, descriptive geometry, fabrication-aware design

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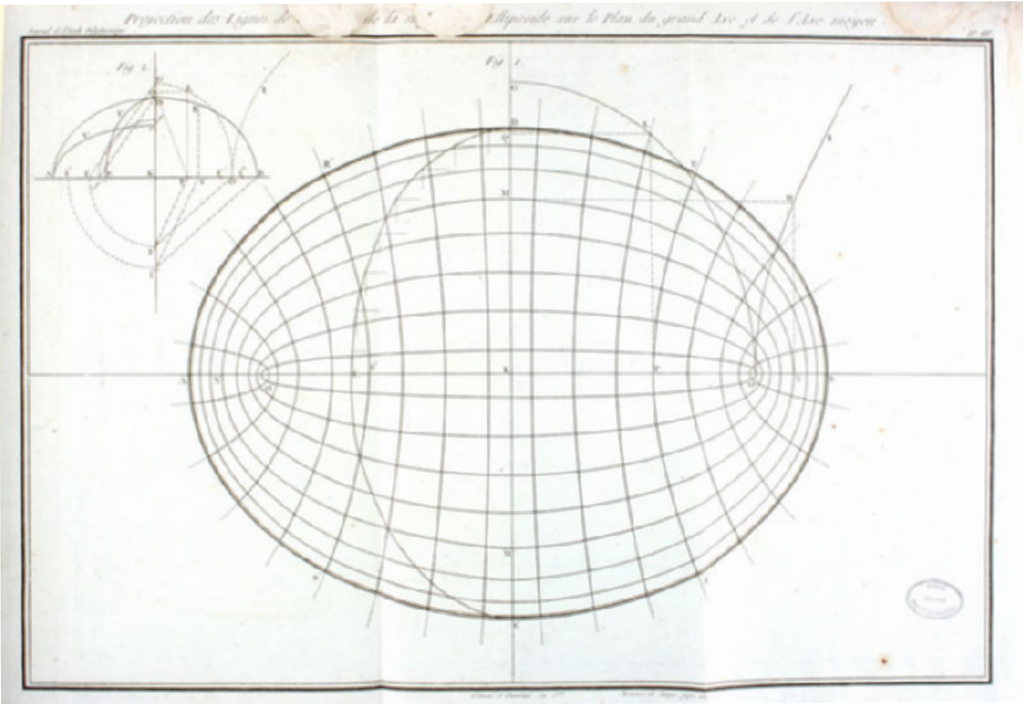

Figure 1. Lines of curvatures of an ellipsoid with descriptive geometry (Leroy 1857).

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1. Introduction

The design of complex architectural shapes has benefited from great advances within the computer graphics community in the last decade. For instance, significant efforts were made to develop numerical methods for the covering of free-form surfaces with planar panels. These methods differ from the common knowledge of architects and engineers, making them hard use for non-specialists to use. The technique proposed in the present article aims thus at bridging this gap with a method that takes inspiration from descriptive geometry, a tool used by architects for centuries, and turns it into a real-time design tool for PQ-meshes.

1.1 Prior Works

Geometrically-Constrained Approach

Planar quadrilaterals have been identified by practicians as an important optimisation target for the construction of double-curved surfaces, as they avoid using curved panels (Glymph et al. 2004). Previous research identified the need for integration of geometrical constraints within the design tools themselves and proposed methods for shape generation of PQ-meshes (Schmiedhofer 2010). Several techniques for generating exact planar quadrilateral meshes were proposed, mostly relying on affine transformations, which preserve planarity, a notion illustrated in Pottmann et al. (2007). For example, *scale-trans surfaces*, introduced in Glymph et al. (2004) use composition of two affine transformations: translation and homothetic transformations. The designer control the shape with two curves, making the process highly intuitive. Despite formal limitations, these shapes have been used in many projects.

Constrained geometric approaches use shapes that are well known and can be rationalised efficiently, for example, towards a high repetition of nodes or panels (Mesnil et al. 2015). They suffer however from a lack of flexibility and form a restricted design space. This led to the introduction of post-rationalisation strategies in order to cover arbitrary shapes with planar quadrilaterals (Liu et al. 2006).

Optimisation-Based Shape Exploration

Most recent methods propose hence to explore design space of feasible solutions for a given mesh topology with the help of optimisation techniques (Deng et al. 2015; Yang et al. 2011). The mesh is interactively deformed by the user with the help of control handles. The overall smoothness is checked with discrete functions of the vertices. To go further, an efficient solver handling quadratic constraints was presented in Tang et al. (2014) and used in Jiang et al. (2014). Projections and subspace exploration are efficiently used for constrained-based optimisation in Bouaziz et al. (2012), Deng et al. (2013, 2015). These methods provide great design freedom, but illustrations shown in the cited references are local deformations

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of meshes. Design space exploration with exact PQ-meshes was also proposed by composition of compatible affine maps assigned to each mesh face and allowed for handle-driven shape exploration (Vaxman 2012). This strategy was extended to other maps that preserve facet planarity by construction in Vaxman (2014).

The idea of this paper is to use the notion of projection, which is commonly used in architecture, especially with plane view and elevations, and to link subspace exploration techniques with representation techniques based on projections in architecture.

Descriptive Geometry

Descriptive geometry is a technique of shape representation invented by the French mathematician Gaspard Monge (Monge 1798; Javary, 1881). It is based on planar orthogonal projections of a solid. The planes in which the projections are done are usually the horizontal and vertical planes. Figure 1 is a typical drawing of descriptive geometry: It describes an ellipsoid with a plane view, displayed with some elevations. The curve network corresponds to the horizontal projection of lines of curvature (Leroy 1857).

Because architectural objects have to deal mainly with gravity and vertical forces, it makes naturally sense to separate projections in vertical and horizontal planes. The idea to use these projections to guide structural design was used recently in the framework of the thrust network analysis, where compression-only structures are found from a planar network at equilibrium (Rippmann et al. 2012; Miki et al. 2015). The objective of this paper is to show that descriptive geometry can be turned into a general tool for the design of PQ meshes and their structural optimisation. The method, called the *Marionette method*, is presented in Section 2, where the relationship between smooth and discrete geometry for PQ-meshes is explained. Section 3 explores then some applications in architecture. Section 4 shows finally the generality of the proposed method, which can be extended to meshes other than the regular quadrilateral meshes and therefore constitute a promising versatile tool to integrate intuitively fabrication constraints into architectural design.

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2. Marionette Meshes

2.1 Marionette Quad

The principles of descriptive geometry can be transposed to architectural shape modelling. The use of appropriate projections provides a simple interpretation of the problem of meshing with flat quadrilaterals. For simplification, we discuss the case of a projection in the (XY) plane in this section; the generalisation to other projections is illustrated in Section 4.

Consider first Figure 2: four points have a prescribed plane view $\mathbf{ABCD}$ in the horizontal plane (P_1). Three points $\mathbf{A}'$, $\mathbf{B}'$, and $\mathbf{D}'$ have prescribed altitudes z_A, z_B, and z_D. In general, there is only one point $\mathbf{C}'$ with the imposed projection $\mathbf{C}$ so that $\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}'$, $\mathbf{D}'$ is planar.

The planarity constraint reads:

(1)

$$\det\left(\mathbf{A}'\mathbf{B}', \mathbf{A}'\mathbf{C}', \mathbf{A}'\mathbf{D}'\right) = 0$$

Expressing coordinates in a cartesian frame of (P_1), and writing $d_{BC} = det\,(\mathbf{AB}, \mathbf{AC})$, $d_{BD} = det\,(\mathbf{AB}, \mathbf{AD})$ and $d_{DC} = det\,(\mathbf{AD}, \mathbf{AC})$, if the points A, B, and D are not aligned, then, one gets:

(2)

$$(z_C - z_A) = \left(\frac{d_{BC}}{d_{BD}}\right) \cdot (z_D - z_A) + \left(\frac{d_{DC}}{d_{BD}}\right) \cdot (z_B - z_A)$$

Figure 2 shows vertical lines used for construction, recalling the strings of a marionette, which gives the name *marionette quad*. Note that the system is underconstrained if the points A, B, and D are aligned, which corresponds to vertical a quad. A projection in the horizontal plane thus allows only for the modelling of height fields. This limitation can be overcome by using other projections (see Section 4).

2.2 Regular Marionette Meshes

Consider now a quadrangular mesh without singularity as depicted in Figure 3. The plane view in the horizontal plane is fixed, and the altitude of two intersecting curves is prescribed. Then, provided that the planar view admits no 'flat' quad (i.e. quad where three points are aligned), equation (2) can be propagated through a strip, and by there through the whole mesh. Indeed, on the highlighted strip

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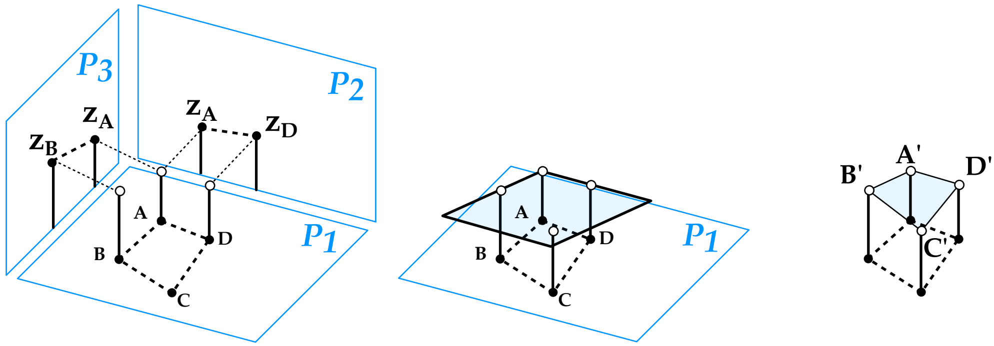

Figure 2. Creation of a Marionette Quad with a plane view and two elevations.

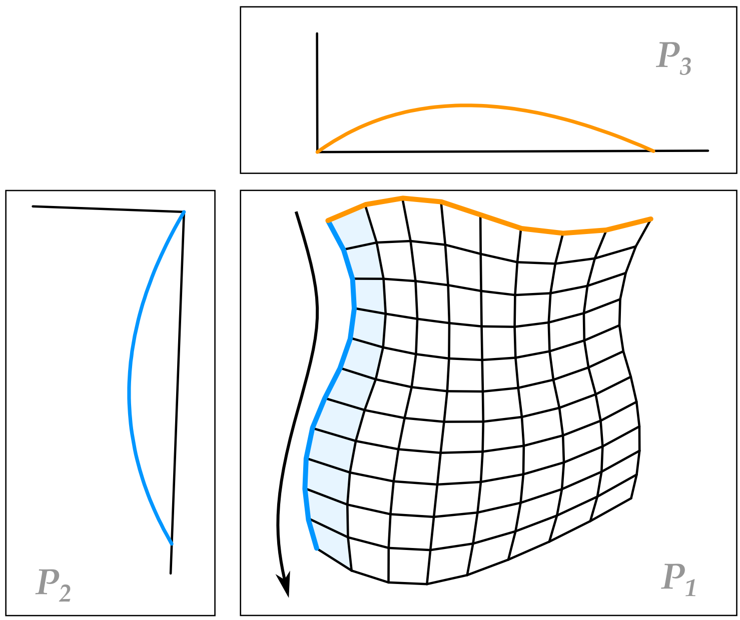

Figure 3. Two elevations and a planar view define a unique Marionette Mesh.

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of Figure 3, the first quad (top left) has three prescribed altitudes, and equation (2) can be used. The same applies for all the quads of the strip.

For a $N \times M$ mesh, the propagation requires $N M$ applications of equation (2), the memory is $3N M$. The marionette technique guarantees hence that the number of operations varies linearly with the number of nodes within a structure. The method performs thus in real time even for meshes with thousands of nodes, as discussed in Section 3.1.

2.3 Link with Smooth Geometry

The proposed method has some interesting relations with smooth geometry. The problem of covering curved shapes with planar panels is linked with the integration of *conjugate curves networks* (Liu et al. 2006; Bobenko & Suris 2008). Conjugate networks correspond to parameterisations (u, v) satisfying the following equation (Bobenko & Suris 2008):

(3)

$$\det\left(\partial_{\mathbf{u}}\mathbf{f}, \partial_{\mathbf{v}}\mathbf{f}, \partial^{\mathbf{2}}_{\mathbf{uv}}\mathbf{f}\right) = 0$$

Consider now that the components in x and y are fixed, as in the problem solved by the Marionette technique. We are looking for the height functions f^z satisfying equation 3. Adopting the notation f_u to denote differentiation of f with respect to u, equation (3) is reformulated into:

(4)

$$\det\begin{pmatrix} f^x_u & f^x_v & f^x_{uv} \\ f^y_u & f^y_v & f^y_{uv} \\ f^z_u & f^z_v & f^z_{uv} \end{pmatrix} = 0$$

Equation (4) is defined if the parameterisation in the plane $(X Y)$ is regular, which means if the study is restricted to height fields. An expansion of the determinant shows that the equation is a second-order linear equation in $f^z(u, v)$. The only term of second order is f^z_{uv} : the equation is thus *hyperbolic*.

Hyperbolic equations often correspond to the propagation of information in a system (think of the wave equation). It is thus no surprise that the marionette method corresponds to a propagation algorithm. Loosely speaking, it can be shown that solutions of hyperbolic equations retain discontinuities of initial conditions. The smoothness of the shape obtained by the marionette method is thus dependent on the smoothness of the input data (plane view and elevation curves).

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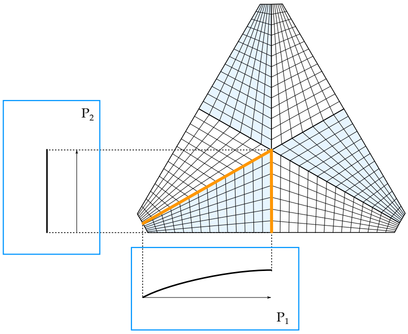
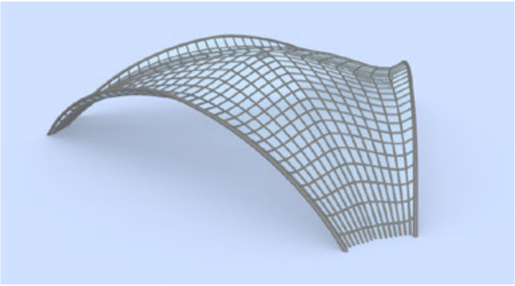
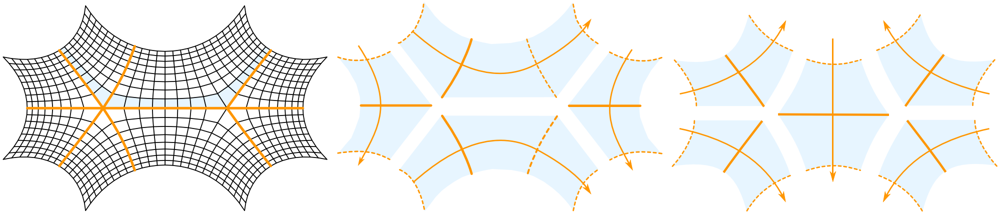


(a) Decomposition of a complex mesh into simple patches.



(b) The corresponding lifted mesh

Figure 4. A Marionette Mesh with a singularity.



(a) Initial mesh (b) Family of four strip-domains (c) Family of five strip-domains

Figure 5. Decomposition of a mesh into 2 families of strip-domains. Marionette Meshes can be generated by choosing one guide curve across each strip-domain.

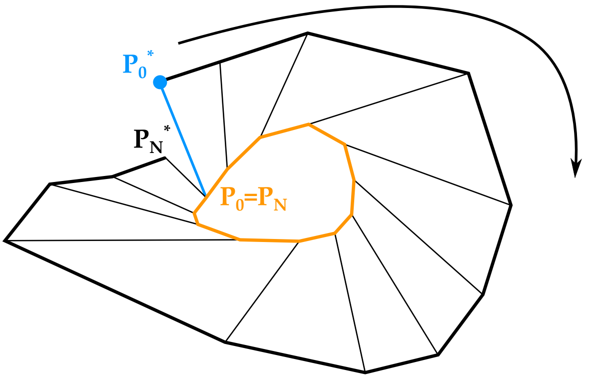


Figure 6. Closed Marionette Strip with incompatible closing condition induced by the prescription of the plane view of the whole strip (orange) and the altitudes of the inner curve (blue).

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2.4 Marionette Meshes with Singularities

The modelling of complex shapes requires the introduction of vertices with a different valence, called singularities in the following. For example, the mesh displayed in Figure 4a has one singularity: the central node has a valence of six. The mesh can be subdivided into six patches with no inner singularity (in blue and white). This kind of procedure can be applied to any quad mesh. Each patch is a regular mesh, and the Marionette technique can be applied. There are, however restrictions on the curves used as guide curves due to compatibility between patches. For example, in Figure 4a, it is clear that the six curves attached to the singularity can be used as guides for the six patches, whereas choosing the 12 curves on the perimeter over-constrain the problem.

For an arbitrary quad mesh, it is possible to compute the number of guide curves that can be used to generate a Marionette Mesh. The mesh can be decomposed into simple quad domains without any singularity, for example, by using the methods described in Tarini et al. (2011) or Takayama et al. (2013). For example, Figure 4a has six domains and the mesh in Figure 5a has nine domains. These domains are four sided, and it is possible to extract independent families of strip domains, like displayed in Figure 5. Depending on the n-colourability of the mesh, the number of families varies. The example showed is two-colourable. As a result, two families of strips can be found and are shown in Figure 5b and 5c. Exactly one curve can be chosen across each strip-domain. Since strips are independent, the height of these nine curves can be chosen independently and will not over-constrain the problem.

2.5 Closed Marionette Meshes

Closed Strips

Marionette Meshes create PQ-meshes by propagation of a planarity constraint along strips. One can easily figure that if the strip is closed, the problem becomes over-constrained. Indeed, consider Figure 6: The plane view of a closed strip and the altitude of the points $(\boldsymbol{P}^i)$ of one polyline are prescribed. If the altitude of the first point used for the propagation $\boldsymbol{P}_0^*$ is chosen, the planarity constraint can be propagated along the strip. The points of the outer line are therefore imposed by the method, and the designer has no control on them. The last point $\boldsymbol{P}_0^*$ is therefore generally different from the initial point $\boldsymbol{P}_0^*$, leading to a geometrical incompatibility of PQ-meshes.

In the following, we develop a strategy to deal with the geometrical compatibility of closed strips. The results, however, can then be extended to general Marionette Mesh with closed strips. Suppose that the two prescribed curves are defined as the inner closed curve and one radial curve (see Figure 6). By propagation of equation (2), we easily see that the altitude of the last point z_N^* depends

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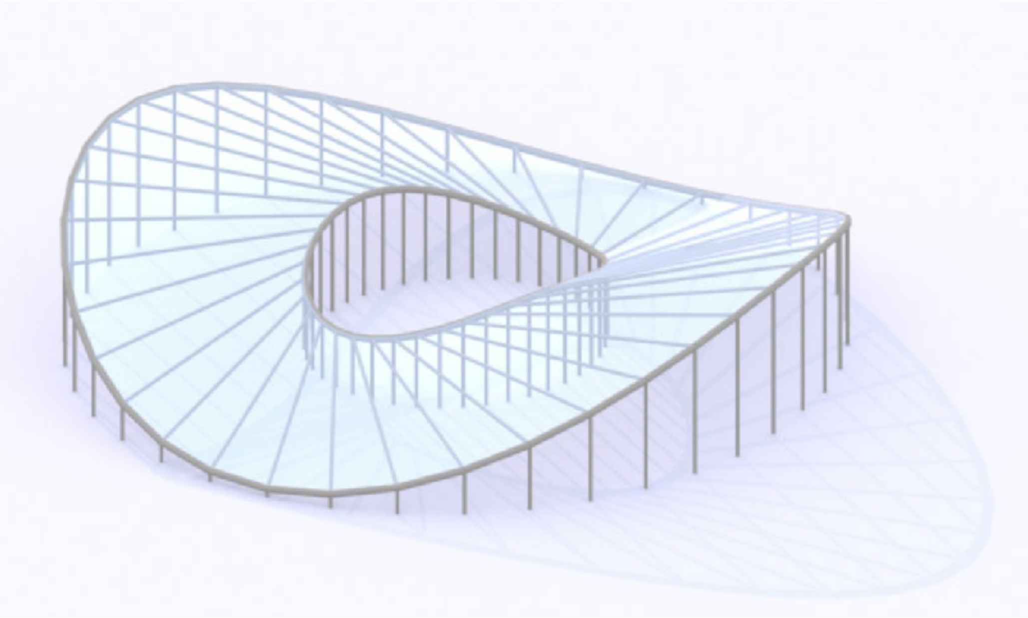

Figure 7. Architectural design with a closed Marionette Mesh, the altitude of the inner curve is prescribed, the designer does not have control on the outer curve.

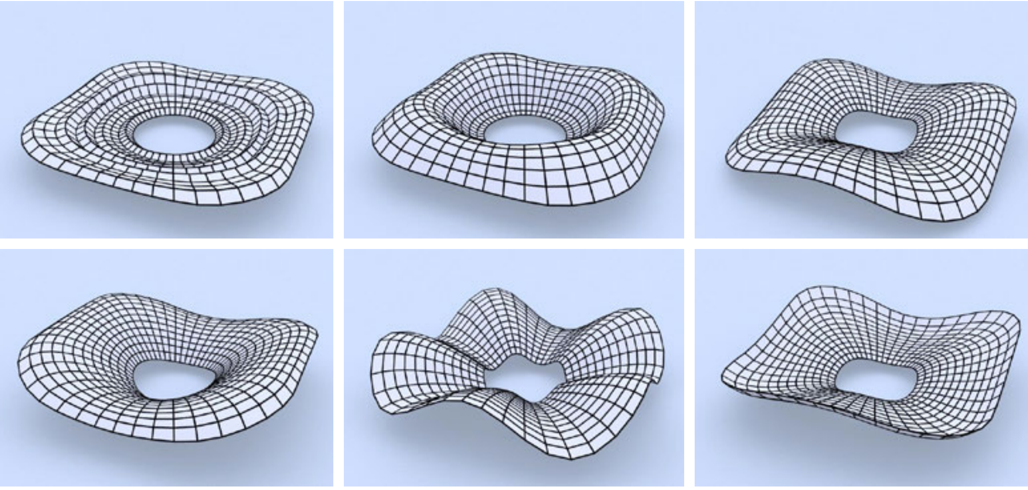

Figure 8. Some shapes with planar faces and a closed mesh generated with the method proposed in this paper.

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linearly on the altitude of the first point z_0^* and on the altitudes of the points on the inner curve $\mathbf{Z}$. It also depends on the in-plane projection of the strip. Formally, there exists a vector $\mathbf{V}$ and a scalar a, both functions of the plane view so that:

(5)

$$\mathbf{V} \cdot \mathbf{Z} + a \cdot z_0^* = z_N^*$$

We are interested in the case where $z_0^* = z_N^*$. There are two possibilities:

1. $\mathrm{a} = 1$, in this case, the condition restricts to $\mathbf{V} \cdot \mathbf{Z} = 0$ and does not depend on z_0^*. The vector $\mathbf{z}$ is in the hyperplane of $\mathbf{V}$, which leaves $N - 1$ degrees of freedom.

2. $\mathrm{a} \neq 1$: there is only one solution for z_0^*. This is the most constrained case: the designer can only control the inner curve of the strip.

Closed Meshes

The meshes with one solution are less flexible, but they can still generate interesting shapes, like the one displayed on Figure 7, which recalls the examples of Figure 6. The designer has a total control on the altitude of the inner curve and the plane view, but cannot manipulate freely the outer curve. Note that the strings of the marionette are here materialised as columns in the rendering, illustrating the geometrical interpretation of the method.

The most interesting case occurs when the designer has potentially the control of two curves. This relies on a condition on the planar view explained above. A simple case where this condition is fulfilled is when it has a symmetry. In this case, there is a $N-1$ parameters family of solutions for the altitude of the inner curve. The elevation of a closed guide curve can be chosen arbitrarily and projected into the hyperplane of normal $\mathbf{V}$, keeping the notations of equation (5). This operation is straightforward and allows one to control the elevation of a second curve, like for open meshes. An example of this strategy is displayed in Figure 8, where all the meshes have the same planar view.

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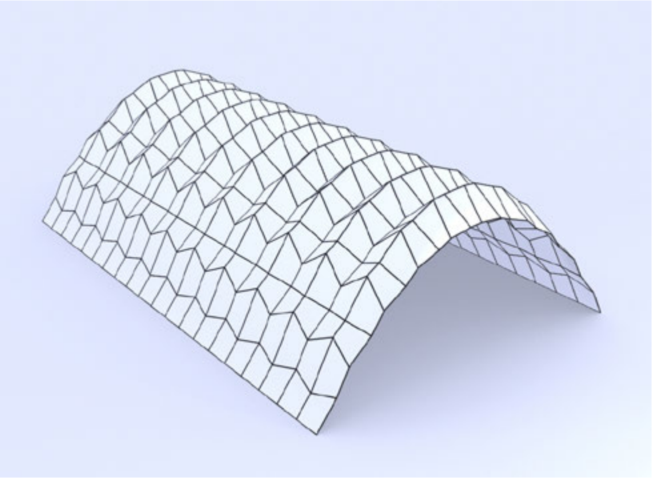

Figure 9. A non-smooth mesh with planar facets generated with the Marionette method.

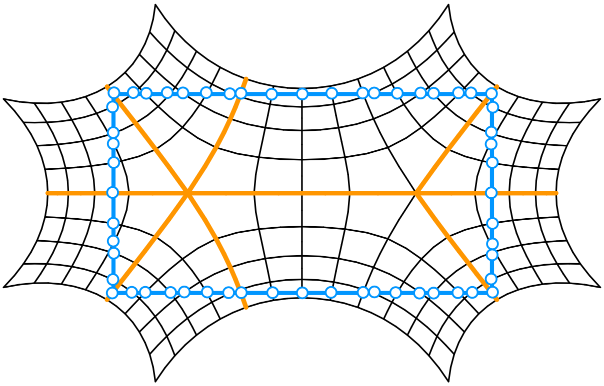

Figure 10. A plane view (thin lines) with a prescribed boundary (thick blue lines).

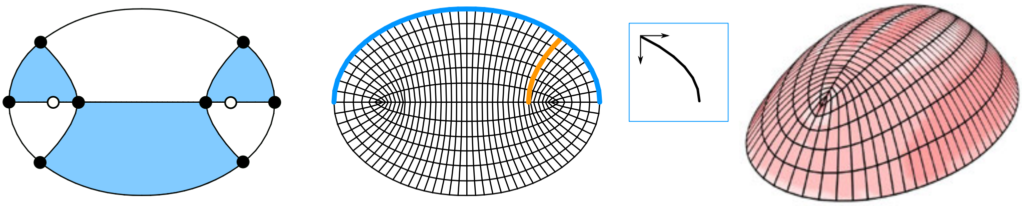

Figure 11. A result of an optimisation procedure: the shell structure is a Marionette Mesh (top view and prescribed curves on the middle) minimising total elastic energy. On the right: red areas indicate compression.

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3. Architectural Design with Marionette Meshes

3.1 Computational Set-up

The algorithms described in this paper have been implemented in the visual-scripting plug-in GrasshopperTM for the modelling software RhinoTM. This allows interaction with other numerical tools necessary for architectural design, like finite-element analysis software. An example of interaction between fabrication-aware shape generation and structural analysis is shown in Section 3.3.

Marionette Meshes only require the solution of a linear system. The computation time is thus low; typically, it takes 3 ms to lift a mesh of 10,000 faces, with no pre-factorisation involved. Real-time computation provides great design flexibility, even for large meshes.

In our framework, the planar views are generated with NURBS patches, and the elevation curves are drawn as Bézier curves. The smoothness of the final mesh depends thus on the smoothness of the in-plane parameterisation. A C^0 projection yields a C^0 solution to the hyperbolic equation (4), so that shape functions with creases can easily be propagated through the mesh. Figure 9 shows a corrugated shape generated from a C^0 planar view and smooth guide curves. Such corrugations can be used in folded plate structures, and could extend the formal possibilities of methods developed in Robeller et al. (2015).

3.2 Shape Exploration with Marionette Meshes

The framework introduced here intrinsically accounts for the planarity of panels. Its mathematical formulation is, however, suited for many architectural constraints. Hard constraints must be fulfilled exactly, whereas soft constraints are included into the function to minimize (Nocedal & Wright, 2006). Since the planarity constraint is linear, soft constraints expressed as linear or quadratic functions can easily be included in the objective function. In this case, the optimisation problem will be similar to a classical least square problem and can be solved efficiently.

Hard constraints defined by linear equations are treated effectively within the proposed framework. Examples of linear constraints are prescribed volume or a maximal allowable altitude. The marionette method imposes $NM-(N+M-1)$ out of NM parameters, this means that another $N+M-1$ linear constraints can be applied without over-constraining the optimisation problem.

Perhaps the most common application of hard constraint in architecture is the prescription of a boundary, as depicted in Figure 10. In this figure, the planar view is imposed and the user prescribes the altitude of some points of the mesh along a curve (white circles). In this case, the number of prescribed points is superior to

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the number of degrees of freedom, and the problem might be over constrained. It might hence be preferable to turn this problem into a soft constrained problem with a quadratic function to minimize. In the same way, for really complex shapes with many singularities or highly constrained boundary, other methods will probably be more efficient, more relevant, and maybe more intuitive, like for example Jiang et al (2014).

3.3 Case Study: Fabrication-Aware Structural Optimisation

The formal possibilities offered by Marionette Meshes are broad enough to offer an interesting design space for engineering problems. Among them, structural optimisation is a particularly relevant. The quick generation of a parameterised design space and the coupling with advanced analysis software seems particularly promising (Preisinger & Heimrath, 2014). Indeed, non-linear criteria, like the buckling capacity, are of high importance for practical design of thin shell or grid shells (Firl & Bletzinger, 2012).

An illustration of the potential of Marionette Meshes for a structurally informed architectural design is proposed in Figure 11: The shell is a Marionette mesh spanning over an ellipse. The plane view is inspired by Figure 1. The mesh is constituted of six NURBS patches and has two singularities (white dots in the image); guide curves are found with the method proposed in this paper. The boundary curve is constrained in the horizontal plane (blue curve on Figure 11). One curve in the other direction (orange curve in Figure 11) defines the whole elevation of the dome. The shell is submitted to gravity load. All the translations at the outer boundary are restricted, and rotations at the supports are allowed (hinges). The model is computed with Finite Element software Karamba3D™. The shape generation of a 1000 faces mesh requires less than 1 ms with the Marionette technique, far less than the assembly and computation of a shell model with FEM.

The structure is optimised towards a minimum of the total elastic energy by means of genetic algorithms. The design parameters are the four altitudes of the control points controlling the shape of the guide curve. It is noticed that tension areas, depicted in blue in Figure 11, are almost non-existent on the inner and upper face of the shell. Hence, if defined properly with an accurate number of singularities, the design space offered by Marionette Meshes is wide enough to find compression-dominant shapes by the means of structural optimisation.

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4. Generalisation of the Method

4.1 General Projections

It appeared that prescribing a horizontal view and applying the propagation technique presented here only allows for the modelling of height fields. This is a limitation of this method, although height fields surfaces are commonly used for roof covering. Other projections can be used for more shape flexibility. The planarity constraint for a quad can be extended to the case of non-parallel projections, like in Figure 12.

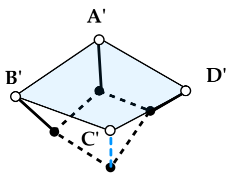

Figure 12. A Marionette quad with non-parallel guide lines.

Some projections are of practical interest for archetypal projects. Towers and facades can be modelled with cylindrical projections. Stadia can be designed using projections on torus or on moulding surfaces, the offset directions corresponding to the normals of the smooth surface. Moulding surfaces fit naturally the geometry of stadia (see Figure 13a) and have some interesting features, discussed in Mesnil et al. (2015):

- Their natural mesh contains planar curves, which are geodesics of the surface: The planarity is preserved by the marionette transformation.
- They are naturally meshed by their lines of curvatures, which gives a torsion-free beam layout on the initial surface, and on the final shape.

4.2 Extension to Other Patterns

The method proposed in this paper can be extended to other polyhedral patterns. As noticed by Deng et al. (2013), tri-hex meshes (also known as Kagom lattices) have the same number of degrees of freedom as quad meshes. There is therefore a straight forward way to lift Kagome lattices with the marionette technique. Figure 14a shows the guide curves for the Kagome pattern. Other isolated points are required to lift the mesh. The altitude of these points can be chosen in order to minimise the fairness energy introduced in Jiang et al. (2014), which is not difficult under linear constraints. Figure 14c shows a pattern introduced in Jiang et

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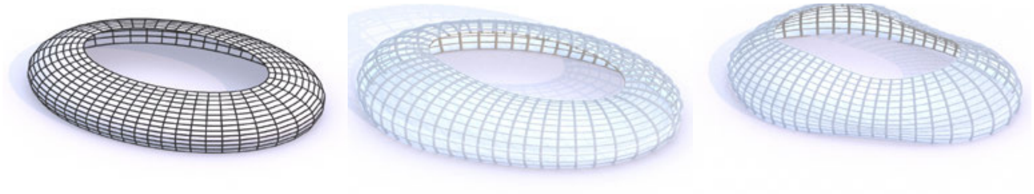
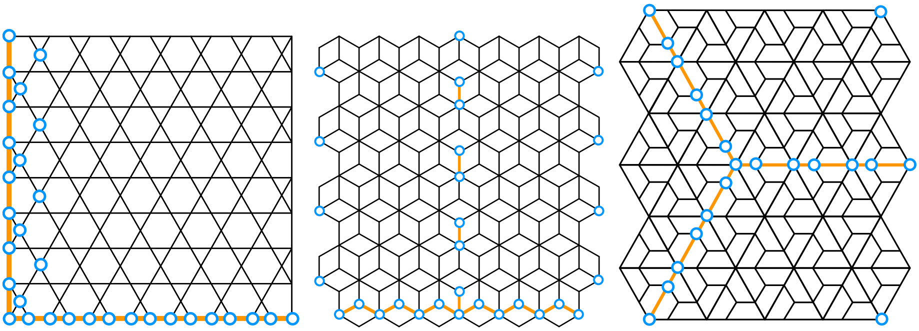


(a) Reference moulding surfaces (b) Non-symmetrical design (c) Symmetrical design

Figure 13. Design of stadia obtained from a projection on a moulding surface: the prescribed curves are the inner ring and a section curve.



(a) Kagome lattice (b) Dual Kagome lattice (c) Hex pattern

Figure 14. Marionette method applied to several patterns, white dots correspond to prescribed altitudes.

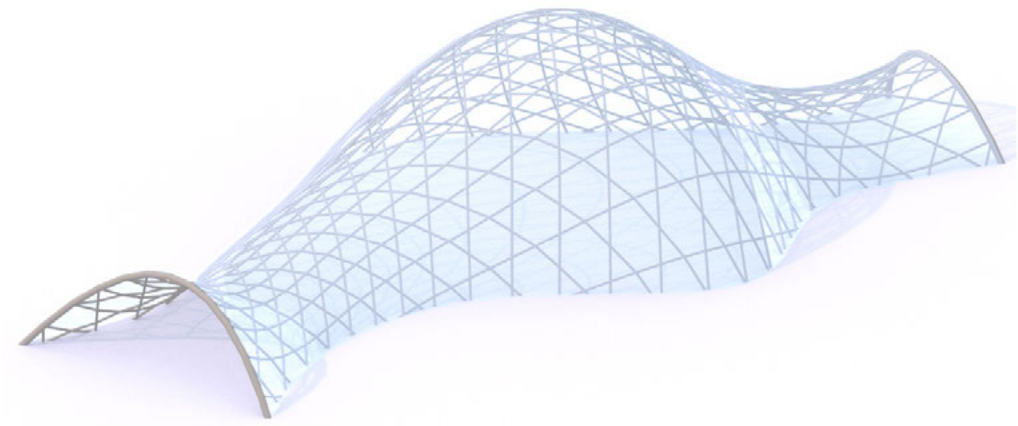


Figure 15. Free-form design covered by planar Kagome lattice.

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al. (2014): The mesh is derived from an hexagonal pattern and three guide curves can be used to lift the mesh.

For example, Figure 15 shows a Kagome lattice covered with planar facets generated with the marionette method. The design started from a planar view generated with a NURBS patch, a Kagome was then generated following the isoparametric lines and lifted with the marionette technique. One of the guide curve is the parabolic arch of the entrance, the other is an undulating curve following the tunnel. Like for PQ-meshes, the computation is done in real time.

5. Conclusion

We have introduced an intuitive technique for interactive shape modelling with planar facets. It is based on descriptive geometry, which is used by architects and engineers. The concept has many applications, in particular the modelling of PQ meshes with or without singularity. Some examples show the formal potential of our method. The framework was also extended to Kagome and dual-Kagome lattices. It is likely that other polyhedral patterns can be treated with the Marionette technique. The generality of the method has also been demonstrated by changing the projection direction, a method with large potential if used on mesh with remarkable offset properties. The choice of appropriate projections, while obvious for many shapes of relatively low complexity, is a limitation to the generality of the method compared to previous methods developed in the field of computer graphics. The Marionette technique should be seen as an intuitive way to model shapes, and is complementary with other less-intuitive methods that perform well on surface-fitting or local exploration problems.

We made a comment on the smooth problem solved by the method, which gives indications on the smoothness of the shapes arising from this framework. We have seen that this smoothness depends on the smoothness of both the planar projection and the guide curves, which can be generated with any usual modelling tool based on NURBS, T-spline and Bézier curves. Moreover, it was shown that marionette meshes give an intuitive illustration on the principle of subspace exploration, a powerful tool for constrained optimisation of meshes. The underlying smooth parameterisation of marionette meshes could hence open new possibilities for efficient parameterisation of fabrication-aware design space in structural optimisation problems.

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